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Set-theoretic Models for Distributive Entailments

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Abstract

The notion of entailment is central to any theory of meaning. If we are told that 'The ball is large and red' is a true sentence, we can be certain that 'The ball is red' is also a true sentence. The first sentence entails the second, which is to say that thereare no circumstances in which the first sentence can be true and the second false.

In this study I consider particularly the sort of entailment found in (1) but not in (2) below.

(1) Two boys slept. (Distributive)

(2) Two boys made a good team. (Collective)

(1) entails 'One boy slept', but (2) does not entail 'One boy made a good team'. The sentence

(3) Two boys ate three cakes.

has both distributive and collective readings (the boys ate three cakes each, or the boys ate three cakes between them).

I attempt to account for these different entailments in a set-theoretic model. Using a simple phrase-structure syntax for a small fragment of English, I consider a series of models, developing the semantic theory until I arrive at a grammar which suggests that the distributive reading of (3) for example would be semantically and syntactically distinct from the collective reading.

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KEY WORDS

entailment	set-theoretic model
distributive	phrase-structure syntax
collective	semantic theory

1.INTRODUCTION

The notion of entailment is central to any theory of meaning. If we are told that ' The ball is large and red' is a true sentence, we can be certain that 'The ball is red' is also a true sentence. The first sentence entails the second, which is to say that there are no circumstances in which the first sentence can be true and the second false.

If we are told though that 'The ball is green and red' is a true sentence, we may be less inclined to say that 'The ball is red' has to be a true sentence, and yet the structures of the two pairs of sentences appear to be similar. How do we explain our different intuitions about the entailments? We may decide we are dealing with differences of meaning (semantics) or differences of structure (syntax). In what follows, I shall be particularly concerned with the entailments which arise from sentences with plural noun phrases (NPs) as their subjects. If we are given

(1) The boys slept.

and we know that Harry is one of the boys, we can be sure from (1) that 'Harry slept' is a true sentence. This is because we know that sleeping involves individual consciousness. It is an act which can only be performed by individuals, although more than one individual may perform it at a time. If the boys slept, and Harry was one of the boys, then necessarily Harry slept.

We cannot similarly deduce from

(2) The boys ate three cakes.

that Harry ate three cakes, since the eating of three cakes may be the action of an individual, or the action of a group. Perhaps each of the boys ate three cakes, or perhaps they ate three cakes between them.

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Finally, given the sentence

(3) The boys made a good team.

(adapted from Roberts 1987 page 6), we do not assume that '?Harry made a good team' has to be a true sentence, since we know that the act of making a team (in the sense intended here of 'constitute a team' rather than, for example, 'manage to become a member of a team') can only be performed by a group, and not by an individual.

The three sentences (1), (2) and (3) tell us about the activities of a particular group of boys, but do not allow us to draw similar conclusions about the actions of any single member of that group.

In the first sentence the action of sleeping is understood to be distributive which is to say that if the boys slept, then necessarily each individual boy slept. In the third sentence the action is collective, since a collection of individuals is required to make a good team and no single individual could do this alone. In the second sentence, we need more information before we are able to say whether the action is distributive or collective. This sentence has both distributive and collective readings.

How can we explain our intuitions about these sentences? The approach which I shall follow here, known as model-theoretic semantics, attempts to relate natural language expressions to abstract entities in a structure known as a model. The models which I shall describe are set-theoretic, and relate natural language expressions to abstract entities regulated by a well defined set theory. Each natural language expression has a corresponding denotation ('meaning' or 'representation') in the model.

No claim is made about the psychological reality of the models discussed here. Indeed there are many types of possible model besides the set-theoretic models found below (see for example Link's 'lattice-theoretical' model, Link 1983). The point of a model is that it allows sometimes little understood facts of natural language to be considered within the relatively well understood framework of the model theory, providing in the process insights into how natural language works.

In what follows, my main interest will be to arrive at a set-theoretic model able to account for the distributive and collective readings found in sentences such as (1), (2) and (3) above. I shall consider in some detail a number of different classes of set-theoretic model for the semantics of a small fragment of English.

2. SEMANTIC MODELS

(i) Singular NPs - The Model M_1

As a starting point, I shall consider the grammar of a fragment of English containing only singular NPs, based on Montague's The Proper Treatment of Quantification in Ordinary English (Thomason 1974 pages 247-270 - henceforth 'PTQ'), but with certain modifications. Firstly, I shall ignore intensions, which I do not believe to be relevant to present concerns. This will allow me to consider a simplified version of the semantics. Secondly, I shall adopt the changes proposed by Bennett in his first fragment, which dispenses with Montague's 'individual concepts' (Bennett 1974 pages 37 and 49-52). Other points on which I diverge from PTQ will be discussed as I proceed.

I shall not give here a full exposition of the syntax or semantics given in either of the works just mentioned, but I shall refer to and explain parts of those grammars as the need arises.

The basic building blocks of PTQ's set-theoretic semantics are the denotations of common nouns and intransitive verb phrases. In Bennett's modified semantics, these are both taken to be sets of individuals. The denotation of the common noun 'man' and the denotation of the VP (verb phrase) 'is a man' would be the same, a set containing every individual who is a man. The denotations of all other syntactic categories are derived in clearly defined ways from these basic denotations. The denotation of a sentence is taken to be either 1, representing the truth of that sentence relative to a particular model, or 0, representing the falsity of that sentence relative to the model.

If we imagine a universe containing three individuals, a, b and c, then the denotation in some model of the common noun 'man' might be {a,c} and the denotation of the common noun 'boy' might be {b} (a is a man, c is a man and b is a boy) [1]. The denotation of the VP 'slept' might be {a,b} while the denotation of 'ate three cakes' (which for the moment I assume to be a basic expression with no internal structure) might be {a,c} (the individual a slept and the individual b slept, the individual a ate three cakes and the individual c ate three cakes).

In Bennett's fragment, an NP denotation is a function from VP denotations to truth values. If, as in Bennett's first fragment, we include only a single tense[2], then effectively the meaning of an NP such as 'one man' is the set of all of those properties which hold of one man. Technically, the denotation of an NP is the characteristic function of a set of VP denotations, which maps every VP denotation onto a truth value. In the model used as an example above, it is true that one boy slept but it is not true that one boy ate three cakes, so the function which is the denotation of 'one boy' will map {a,b} (the denotation of 'slept') onto 1 (i.e. true), but it will map {a,c} (the denotation of 'ate three cakes') onto 0 (i.e. false).

If common noun denotations are sets and NP denotations are functions from sets to truth values, then determiner denotations[3] have to be the most complex functions we have considered so far. A determiner denotation must map every common noun denotation onto an NP denotation, so it must be a function from sets (common noun denotations) to functions from sets (VP denotations) to truth values.

As I shall propose certain modifications in order to include a distinction between distributive and collective NPs, let me set out more formally the simple model I have suggested.

A distinction must be made between the model itself and the semantic theory of which it is an example. Given a particular theory of the semantics of a language, there are many models which would be compatible with that theory. Indeed, the theory itself might be seen as a definition of a set of possible models. Dowty, Wall and Peters (1981) make the following distinction.

> Formally, a model is an ordered pair <A,F>, where A is a set, the set of individuals, and F is a function which assigns semantic values of the appropriate sort to the basic expressions. All the rest (for which there seems to be no standard name in the literature) is taken as the fixed part of the semantics for a particular language...
>
> (page 45)

For the model discussed above, A would be the set of individuals {a,b,c} and F would be a function which assigned denotations to basic expressions, such as the set {a,c} as the denotation of the common noun 'man'. 'All the rest', as Dowty et al. put it, might for example include the theory-specific conditions that common nouns and verb phrases should always have denotations which are sets of individuals, and that NP denotations should be functions from VP denotations to truth values.

Many different models would be compatible with such a semantic theory, and we can obviously change the model without changing the semantic theory. In what follows, I shall consider a number of different models as examples of the type of model available within a particular semantic theory. I shall also consider changes to the semantic theory.

To put our simple model in the context of a complete grammar, let us assume a simple phrase-structure syntax.

S → NP VP

NP → Det CN

The basic syntactic categories of this grammar are VP, Det (determiner), and CN (common noun), since no rule generates these categories out of more basic categories. (I am assuming for this grammar that VP is a basic category, with no internal structure.)

The members of these basic syntactic categories, the basic expressions of the language, we shall take to be

VP → {slept, ate three cakes}

Det → {one}

CN → {man, boy}

A function F assigns a denotation to each of the basic expressions of the language. There are many possible types of model compatible with this simple language, and many possible models of each type. For each model, a different function F will assign denotations to basic expressions. If we index the model under consideration as M_1, we can index as F_1 the function which assigns a denotation to each of the basic expressions of the language in that model.

Just as the phrases of the language are generated via the syntax from the basic expressions, so the denotations of those phrases are derived via the model from the denotations of the basic expressions of which they consist. We need some formal statement of the way the denotations of basic expressions combine to give denotations of phrases and, in the manner of PTQ, we might do this by giving a semantic rule to accompany each syntactic rule.

Syntactic Rule 1: S → NP VP

Semantic Rule 1: If a member of category NP has a denotation α and a member of category VP has a denotation β, and they combine as a member of category S, then the denotation of that member of category S is $\alpha(\beta)$.

Syntactic Rule 2: NP → Det CN

Semantic Rule 2: If a member of category Det has a denotation α and a member of category CN has a denotation β, and they combine as a member of category NP, then the denotation of that member of category NP is $\alpha(\beta)$.

Finally, we might specify function F_1. The denotations assigned by F_1 to the members of the basic syntactic categories would be

$$F_1 = \left[\begin{array}{lll}
\text{man} & \rightarrow & \begin{bmatrix} a \rightarrow 1 \\ b \rightarrow 0 \\ c \rightarrow 1 \end{bmatrix} \\[2ex]
\text{boy} & \rightarrow & \begin{bmatrix} a \rightarrow 0 \\ b \rightarrow 1 \\ c \rightarrow 0 \end{bmatrix} \\[2ex]
\text{slept} & \rightarrow & \begin{bmatrix} a \rightarrow 1 \\ b \rightarrow 1 \\ c \rightarrow 0 \end{bmatrix} \\[2ex]
\text{ate three cakes} & \rightarrow & \begin{bmatrix} a \rightarrow 1 \\ b \rightarrow 0 \\ c \rightarrow 1 \end{bmatrix} \\[2ex]
\text{one} & \rightarrow & \left[\begin{array}{lll}
\begin{bmatrix} a \rightarrow 1 \\ b \rightarrow 0 \\ c \rightarrow 1 \end{bmatrix} & \rightarrow & \left[\begin{array}{ll}
\begin{bmatrix} a \rightarrow 1 \\ b \rightarrow 1 \\ c \rightarrow 0 \end{bmatrix} & \rightarrow 1 \\
\begin{bmatrix} a \rightarrow 1 \\ b \rightarrow 0 \\ c \rightarrow 1 \end{bmatrix} & \rightarrow 1 \\
\vdots & \vdots
\end{array}\right] \\[6ex]
\begin{bmatrix} a \rightarrow 0 \\ b \rightarrow 1 \\ c \rightarrow 0 \end{bmatrix} & \rightarrow & \left[\begin{array}{ll}
\begin{bmatrix} a \rightarrow 1 \\ b \rightarrow 1 \\ c \rightarrow 0 \end{bmatrix} & \rightarrow 1 \\
\begin{bmatrix} a \rightarrow 1 \\ b \rightarrow 0 \\ c \rightarrow 1 \end{bmatrix} & \rightarrow 0 \\
\vdots & \vdots
\end{array}\right] \\
\vdots & &
\end{array}\right]
\end{array}\right]$$

The denotation assigned by F_1 to the determiner 'one', based on the denotation of 'a(n)'[14] in PTQ, has been abbreviated to show only those parts of it which would be of use for the model under consideration. In fact given the semantic theory, the denotation of 'one' would be the same in any model containing three individuals. The three individuals in the model can combine in different ways to give 2^3 sets of individuals. In the model under consideration only two of the eight possible sets of individuals are assigned as denotations of common nouns, but the model allows up to eight common noun denotations. The full denotation of 'one' would map every one of the eight potential common noun denotations onto a function from VP denotations to truth values, regardless of whether those potential common noun denotations were assigned to basic expressions of the language.

Similarly, since VP denotations are also sets of individuals, there are eight possible VP denotations in this model, but only two of them have been assigned to basic expressions of the language. The full denotation of 'one' would map every one of the eight potential common noun denotations onto a function which mapped every one of the eight potential VP denotations onto a truth value, regardless of whether those potential VP denotations were assigned to basic expressions of the language.

Put simply, the denotation of 'one' looks for an intersection between common noun denotations and VP denotations. It takes the input set (the common noun denotation) and matches it against each of the eight VP denotations. If any members of the input set are also members of a VP denotation, then that VP is mapped onto 1, otherwise it is mapped onto 0.

If we consider again the examples

(6) One boy slept.

(7) One boy ate three cakes.

discussed above, we can see in the model that the denotation of 'one' maps the denotation of 'boy' onto a function which maps the denotation of 'slept' onto 1, but the denotation of 'ate three cakes' onto 0.

Horn (1976 pages 31-32) draws a distinction between 'at least one' and 'exactly one', but explains the two interpretations in terms of 'rules of conversation' rather than treating them as a 'purely linguistic ambiguity'. Bennett (1974 page 185) has different denotations for 'at least one' and 'at most one' (which differs from 'exactly one' in that 'At most one boy slept' is true when no boys slept).

In fact the denotation of 'one' in the model above is more precisely 'at least one', as we can see from

(8) One man ate three cakes.

which comes out as a true sentence, even though more than one man actually ate three cakes.

Whatever the status of the 'at least/ exactly/ at most' distinction, and it is not my main concern here, it need not present a problem for the model theory. For example, the function F_1 could assign a denotation to 'exactly one', which would be distinct from the denotation of 'at least one' in that it would look for a single individual and no more in the intersection of the input NP denotation and the VP denotation. Like the denotation of 'at least one man', the denotation of 'exactly one man' would map the denotation of 'slept' onto 1 (one man and only one man slept), but unlike the denotation of 'at least one man', it would map the denotation of 'ate three cakes' onto 0 (two men ate three cakes). Since the denotation of 'boy' contains a single individual, 'exactly one boy' would have the same denotation as 'at least one boy'. 'At most one' could be handled in a similarly straightforward way, by looking for either a single individual and no more in the intersection of the input NP denotation and the VP denotation, or an empty intersection.

Rather than write out in full a set-theoretic model as I have done, both PTQ and Bennett's first fragment represent the denotations of determiners using a separate logical language. This is a more general way of approaching these denotations, since it gives a once and for all translation of the determiners which can be used in all models which use the same type of set-theoretic objects as denotations of syntactic categories. My denotation of 'one', given for a model involving only three individuals, would need to be redefined for any model which contained a different number of individuals. It is possible to look at the model-theoretic denotation of 'one' which is (partially) set out above, and generalize as I have done about the way the denotation is worked out, but the generalization is not an explicit part of the denotation.

Using the translation of 'a(n)' in PTQ (page 261, also Bennett 1974 page 40), the sentence

A man slept.

would translate into

$\exists x(\text{man}'(x) \wedge \text{slept}'(x))$

This claims that there exists an individual who is a member of the set which denotes 'man', and also a member of the set which denotes 'slept'. The sentence is true if such an individual does exist, false if not.

The logical translation of 'a(n)' in PTQ and the model-theoretic denotation of 'one' given above do the same job, in the sense that both relate the truth of a sentence to the existence of an individual who is a member of both the set which denotes 'man', and the set which denotes 'slept'.

(ii) The Model M_2 (Distributive Readings of Plural NPs)

Suppose we wish now to extend the fragment just discussed by including the determiner 'two'. We need to modify the syntax to allow for the plural forms 'men' and 'boys', and we need to include 'two' in the basic expressions of the category of determiners. We may find that such changes cannot be satisfactorily accommodated within the semantic theory put forward so far, but let us assume for the moment that they can. As an example of how this might work, we might extend the model M_1 to include a denotation for the new basic expression 'two'. Let us call this extended model M_2, and the function which assigns denotations to basic expressions F_2.

I shall ignore the syntactic complication of the plural forms, and simply assume that the syntax has unspecified devices for selecting between 'man/men', 'boy/boys'. I shall also initially not address directly the question of distributive and collective readings.

As a starting point, let us assume that F_2 is identical to F_1, except that in addition to the denotations assigned by F_1, F_2 will also assign a denotation to the basic expression 'two'. In this model, 'exactly two' would have the same denotation as 'at least two', since there are no more than two individuals in any common noun denotation. Either denotation would come out as

$$F_2 = \left[\begin{array}{l} \vdots \\ \text{two} \to \left[\begin{array}{l} \begin{bmatrix} a\to 1\\ b\to 0\\ c\to 1 \end{bmatrix} \to \left[\begin{array}{l} \begin{bmatrix} a\to 1\\ b\to 1\\ c\to 0 \end{bmatrix} \to 0 \\ \begin{bmatrix} a\to 1\\ b\to 0\\ c\to 1 \end{bmatrix} \to 1 \\ \vdots \end{array} \right] \\ \begin{bmatrix} a\to 0\\ b\to 1\\ c\to 0 \end{bmatrix} \to \left[\begin{array}{l} \begin{bmatrix} a\to 1\\ b\to 1\\ c\to 0 \end{bmatrix} \to 0 \\ \begin{bmatrix} a\to 1\\ b\to 0\\ c\to 1 \end{bmatrix} \to 0 \\ \vdots \end{array} \right] \\ \vdots \end{array} \right] \end{array} \right]$$

The denotation of 'two' in this model is worked out in exactly the same way as the denotation of 'one' in M_1, except that it looks for at least two individuals who are members of the common noun denotation and also of the VP denotation.

The corresponding logical translation of 'two', of the sort found in PTQ, would be

(9) $\lambda P \lambda Q \exists x,y(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge x \neq y)$

where lower case letters are individual variables and upper case letters are simple predicate variables. This would give us examples like

(10) $\exists x,y(\text{man}'(x) \wedge \text{man}'(y) \wedge \text{ate three cakes}'(x)$
$\wedge \text{ate three cakes}'(y) \wedge x \neq y)$

(There exist two distinct individuals, each of whom is a member of the set which denotes 'man' and a member of the set which denotes 'ate three cakes'.)

If we define a function

(11) $2(f) \leftrightarrow \exists x_1,x_2(f(x_1) \wedge f(x_2) \wedge x_1 \neq x_2)$

then (9) and (10) might be simplified[5] to (12) and (13) respectively.

(12) $\lambda P \lambda Q \exists X(2(X) \wedge \forall y(X(y) \rightarrow P(y) \wedge Q(y))$

(13) $\exists X(2(X) \wedge \forall y(X(y) \rightarrow \text{man}'(y) \wedge \text{ate three cakes}'(y))$

(There exists a set containing two distinct individuals, and every member of that set is a member of the set which denotes 'man' and a member of the set which denotes 'ate three cakes'.)

We might assume (or define) a function similar to (11) for each of the cardinal numbers. The function for 3, for example, would be

(14) $3(f) \leftrightarrow \exists x_1,x_2,x_3(f(x_1) \wedge f(x_2) \wedge f(x_3) \wedge x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge x_2 \neq x_3)$

which is longer, although no more complicated, than the function for 2, mainly because it distinguishes three, not two, individuals one from another.

We can now define a generalized translation of all of the cardinal numbers: replace '2' in (12) by any numeral, to obtain the logical translation of the corresponding English number word. Such a generalization would work just as well for '1' as for other numbers. This would add unnecessary complication to the translation of 'one' (e.g. the notion of every member of a singleton set) but would bring it into line with the translations of 'two', 'three' etc..

Let us consider now a problem with the new model, M_2. Given the semantic theory on which M_2 is based, if (15) is true in any model of this type, then necessarily (8) is also true.

(15) Two men ate three cakes.

(8) One man ate three cakes.

The model-theoretic denotation of 'two men' maps onto 1 any VP denotation which contains at least two individuals from the set which is the denotation of 'man'. It is not possible for any VP denotation to contain at least two such individuals, and yet not contain at least one such individual. Similarly, simplification of the logical translation of (15), given above as (13) and based on (10), also predicts that (8) is true.

This entailment is not what our intuitions might lead us to expect. In the model M2, the NP 'two men' in (15) is distributive with respect to the VP 'ate three cakes'. In fact, if we had a model of this type which included more common nouns and more VPs, it would always be the case that the subject NP was distributive with respect to the VP.

This would be fine for a VP such as 'slept', but for 'ate three cakes' there may well be a situation in which two men ate three cakes between them, rather than eating two cakes each. For the VP in

(3) The boys made a good team.

discussed above, such a model would be inappropriate for the intended collective reading.

The problem that we have with the model M_2 is in fact a problem with the semantic theory. The model is based on the semantic theory we find behind Bennett's first fragment (Bennett 1974), which does not deal with plural NPs or plural VPs, so that the problem of collective and distributive readings does not arise. We cannot solve the problem by substituting for the model M_2 some other model from the class of models presupposed by the same semantic theory. We must now look to modify the semantic theory itself.

(iii) The Model M_3 (Collective Readings of Plural NPs)

In the first two models we have considered, VP denotations have been sets of individuals, following PTQ. The question of distributive and collective readings does not arise in PTQ, nor in the first fragment in Bennett (1974), since both deal only with singular NPs.

If the denotation of 'ate three cakes' is the set {a,c}, then the basic idea is that the individual a ate three cakes and the individual c ate three cakes. We have seen in the previous section that such denotations are inadequate. They only allow for individuals to perform actions, never groups, and so they will always produce readings where the subject NP distributes over the VP.

If we wish to capture the collective reading of a sentence such as

(15) Two men ate three cakes.

so that it is equivalent to

(16) Two men ate three cakes between them.

then we need a different sort of model from the models we have considered so far. We need to change our semantic theory to ensure that the two men are represented in the model as some sort of unit. I shall continue with the sort of set-theoretic model theory of PTQ and assume that the unit we require is a set.

In models M_1 and M_2, 'ate three cakes' has a denotation which is the set of individuals {a,c}, which is identical to the denotation of 'man'[6]. Let us now take the denotations of VPs to be not sets of individuals but sets of sets of individuals.

Since we already have a model which can account for distributive readings (M_2) and the problem appears to be with collective readings, let us consider first of all a new model M_3, to account only for collective readings. M_3 will be an alternative model for the same language as M_2, so we do not need to consider any alteration to the syntax, nor do we necessarily need to change semantic rules 1 and 2, so let us assume that they will stay as they are. To create the new model, let us specify a new function, F_3.

At this point, we inevitably begin to lose sight of the model as a whole. In models M_1 and M_2, where VP denotations are sets of individuals, with three individuals we have 8 possible VP denotations. Now that VP denotations are to be sets of sets of individuals, we have 2^8 possible VP denotations. The function which is the denotation of 'two' must map each common noun denotation onto a function which maps each one of the 256 possible VP denotations onto a truth value.

It is still just about possible to give an impression of what a complex model of this type would look like, by showing the denotations of just one common noun, one VP and that part of the denotation of 'two' which relates the two.

$$F_3 = \left[\begin{array}{lll} \text{man} & \to & \begin{bmatrix} a \to 1 \\ b \to 0 \\ c \to 1 \end{bmatrix} \\ \text{ate three cakes} & \to & \begin{bmatrix} \begin{bmatrix} a \to 0 \\ b \to 0 \\ c \to 0 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 0 \\ b \to 0 \\ c \to 1 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 0 \\ b \to 1 \\ c \to 0 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 0 \\ b \to 1 \\ c \to 1 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 1 \\ b \to 0 \\ c \to 0 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 1 \\ b \to 0 \\ c \to 1 \end{bmatrix} \to 1 \\ \begin{bmatrix} a \to 1 \\ b \to 1 \\ c \to 0 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 1 \\ b \to 1 \\ c \to 1 \end{bmatrix} \to 0 \end{bmatrix} \\ \text{two} & \to & \begin{bmatrix} \begin{bmatrix} a \to 1 \\ b \to 0 \\ c \to 1 \end{bmatrix} \to \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} a \to 0 \\ b \to 0 \\ c \to 0 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 0 \\ b \to 0 \\ c \to 1 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 0 \\ b \to 1 \\ c \to 0 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 0 \\ b \to 1 \\ c \to 1 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 1 \\ b \to 0 \\ c \to 0 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 1 \\ b \to 0 \\ c \to 1 \end{bmatrix} \to 1 \\ \begin{bmatrix} a \to 1 \\ b \to 1 \\ c \to 0 \end{bmatrix} \to 0 \\ \begin{bmatrix} a \to 1 \\ b \to 1 \\ c \to 1 \end{bmatrix} \to 0 \end{bmatrix} \to 1 \\ \begin{bmatrix} \; \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{array}\right]$$

Although it is not obvious from this small corner of the model, if this model is designed to allow collective readings only, then the denotation of 'two men' will map onto 1 only those VP denotations which contain a set which contains two men.

It may be clearer if we use set notation. In the corner of the model set out above, the denotation of 'ate three cakes' is the set of sets {{a,c}} and the denotation of 'man' is the set {a,c}. The function which is the denotation of 'two men' maps {{a,c}} onto 1 because the set of sets {{a,c}} contains the set {a,c}, which contains two men. If only collective readings are to be recognized, then a VP which had the denotation {{a},{c}} would not be mapped onto 1 by the function which is the denotation of 'man', since it contains no set which contains two men.

A logical translation for 'two' corresponding to models of the type we are now considering would be

(17) $\lambda B \lambda \delta \exists Y(2(Y) \wedge \delta(Y) \wedge \forall x(Y(x) \rightarrow B(x)))$

where the lower case Greek letter δ is a variable which represents a set of sets. This would give us examples like

(18) $\exists Y(2(Y) \wedge \text{ate three cakes}'(Y)$
$\wedge \forall x(Y(x) \wedge \text{man}'(x)))$

(There exists a set of individuals which has two members. That set ate three cakes, and every member of that set is a man.)

With this revised semantic theory, (15) does not entail (8) as it did with the semantic theory assumed for the model M_2.

(15) Two men ate three cakes.

(8) One man ate three cakes.

Before, the existence of two individuals who ate three cakes entailed the existence of one individual who ate three cakes. Now we are concerned with the existence of a set of individuals which ate three cakes, and the existence of such a set does not entail the existence of any single individual who ate three cakes.

We now have examples of alternative models, corresponding to alternative semantic theories, for the semantics of the same language. Models such as M_2 can account for distributive readings, and models such as M_3 can account for plural readings. This is essentially the stance taken on distributive and collective readings in the second fragment in Bennett (1974).

The problem with this analysis is that the denotations of NPs and VPs in M_2 are different sorts of objects from the denotations of NPs and VPs in M_3. An NP denotation from M_2 would be incompatible with a VP denotation from M_3. If we decide to keep both types of model, then given an expression of the language, how do we know which model to turn to for the denotation of that expression?

Bennett's solution is to give a syntax for each of two distinct languages, one of which maps onto one type of model to produce distributive readings, the other onto the other type of model to produce collective readings. Of course, there is some overlap between the two languages, and the two are woven together into a single language to give sentences which contain elements from both. Nevertheless there is a high degree of redundancy, as often what would normally be regarded as a single syntactic rule has to be stated twice, once for each of the two separate languages which are embedded in the overall grammar.

(iv) The Model M_4 (Distributive and Collective Readings)

If we wish to avoid a two-tier syntax and semantics, then we must bring into line the sorts of model-theoretic objects which are assigned as denotations of VPs and NPs. VPs for example must either always be sets of individuals, or they must always be sets of sets of individuals, or whatever other type of model-theoretic object is appropriate.

We have seen from the example of M_2 that if we take VP denotations to be sets of individuals, we cannot capture the correct entailments for collective readings. We need VP denotations to be sets of sets of individuals for collective readings. Let us consider now the possibility of capturing distributive entailments with VP denotations as sets of sets of individuals.

We can still retain most of the semantic theory exemplified by the model M_3, but we need to look again at the denotation of the determiner 'two'. Let us construct M_4, an instance of a new type of model, which will include distributive readings as well as collective readings. F_4 will differ from F_3 in three ways. Firstly, M_4 will be a model for the semantics of a slightly different language from M_3. Let us introduce a new VP 'ate three eggs'. This will be included in the basic expressions of category VP and will be assigned a denotation by F_4. Let that denotation be the set of sets of individuals {{a},{c}}.

Secondly, the same denotation which was assigned to 'two' by F_3 will be assigned instead by F_4 to a new basic expression 'two[-DIST]', a member of the category Det, and this category will no longer have as a member the basic expression 'two'.

Thirdly, another new basic expression of category Det, 'two[+DIST]', will be assigned a denotation by F_4.

The syntactic rules and the semantic rules of the language will be the same as for the first language we considered, when M_1 was discussed. The basic expressions of the basic syntactic categories of the new language are

VP → {ate three cakes, ate three eggs}

Det→ {two[-DIST], two[+DIST]}

CN → {man}

(For simplicity, I have not included 'one', 'boy' or 'slept' in this language.)

The function F_4 consists of the denotations assigned by the function F_3, with 'two[-DIST]' assigned by F_4 the denotation which was assigned to 'two' by F_3, plus the following denotations.

$$F_4 = \left[\begin{array}{ll}
\text{ate three eggs} \to & \left[\begin{array}{l}
\begin{bmatrix} a\to 0\\ b\to 0\\ c\to 0\end{bmatrix} \to 0\\
\begin{bmatrix} a\to 0\\ b\to 0\\ c\to 1\end{bmatrix} \to 1\\
\begin{bmatrix} a\to 0\\ b\to 1\\ c\to 0\end{bmatrix} \to 0\\
\begin{bmatrix} a\to 0\\ b\to 1\\ c\to 1\end{bmatrix} \to 0\\
\begin{bmatrix} a\to 1\\ b\to 0\\ c\to 0\end{bmatrix} \to 1\\
\begin{bmatrix} a\to 1\\ b\to 0\\ c\to 1\end{bmatrix} \to 0\\
\begin{bmatrix} a\to 1\\ b\to 1\\ c\to 0\end{bmatrix} \to 0\\
\begin{bmatrix} a\to 1\\ b\to 1\\ c\to 1\end{bmatrix} \to 0
\end{array}\right]\\
\begin{array}{c}\text{two}\\ \text{[+DIST]}\end{array} \to & \left[\begin{array}{l}
\begin{bmatrix} a\to 1\\ b\to 0\\ c\to 1\end{bmatrix} \to \left[\begin{array}{l}
\left[\begin{array}{l}
\begin{bmatrix} a\to 0\\ b\to 0\\ c\to 0\end{bmatrix} \to 0\\
\begin{bmatrix} a\to 0\\ b\to 0\\ c\to 1\end{bmatrix} \to 1\\
\begin{bmatrix} a\to 0\\ b\to 1\\ c\to 0\end{bmatrix} \to 0\\
\begin{bmatrix} a\to 0\\ b\to 1\\ c\to 1\end{bmatrix} \to 0\\
\begin{bmatrix} a\to 1\\ b\to 0\\ c\to 0\end{bmatrix} \to 1\\
\begin{bmatrix} a\to 1\\ b\to 0\\ c\to 1\end{bmatrix} \to 0\\
\begin{bmatrix} a\to 1\\ b\to 1\\ c\to 0\end{bmatrix} \to 0\\
\begin{bmatrix} a\to 1\\ b\to 1\\ c\to 1\end{bmatrix} \to 0
\end{array}\right] \to 1\\
\vdots
\end{array}\right]\\
\vdots
\end{array}\right]
\end{array}\right]$$

The denotation of 'two[+DIST] men' in this model will map onto 1 only those VP denotations which contain two singleton sets of men. If such a VP denotation contains two singleton sets of men, then necessarily it must contain at least[7] one singleton set of men, so in models of this type if (19) is true then (20) must also be true.

(19) Two[+DIST] men ate three eggs.

(20) One man ate three eggs.

We already have a model-theoretic denotation and a logical translation of 'two[-DIST]', given as (17) for 'two' in M_3.

(17) $\lambda B \lambda \delta \exists Y(2(Y) \wedge \delta(Y) \wedge \forall x(Y(x) \rightarrow B(x)))$

The logical translation of 'two[+DIST]' would be

(21) $\lambda Q \lambda \gamma \exists \delta (2(\cup \delta) \wedge \forall B(\delta(B) \rightarrow (1(B) \wedge \gamma(B) \wedge \forall x(B(x) \rightarrow Q(x)))))$

where $2(\cup \delta)$ means that the union of the set of sets of individuals δ contains two individuals. This would give examples like

(22) $\exists \delta (2(\cup \delta) \wedge \forall B(\delta(B) \rightarrow (1(B) \wedge \text{ate three eggs}'(B) \wedge \forall x(B(x) \rightarrow \text{man}'(x)))))$

(There exists a set of sets of individuals, the union of which contains two individuals. Every set of individuals which is a member of that set of sets of individuals is a singleton set of individuals which ate three eggs, and every member of that singleton set of individuals (i.e. the unique member) is a man.)

There are disadvantages to such an analysis. Most obviously, an extension of M_4 would need an ambiguous 'three', an ambiguous 'four', etc.. To account for distributive and collective readings, each plural determiner in the language would need to appear twice in the lexicon, once with the feature [+DIST] and once with the feature [-DIST].

One way around this problem would be to have just 'two' in the lexicon, and have as denotations for [+DIST] and [-DIST] functions which map determiners onto distributive or collective determiners, or else NPs onto distributive or collective NPs.

Rather than having 'two[-DIST]' and 'two[+DIST]' listed separately in the lexicon and each assigned a different denotation in the model, it would be possible to have a version of 'two' which had a denotation which consisted of the union of the denotations of the other two expressions. In a new model, let us call it model M_{4b}, the NP 'two men' for example would map onto 1 any VP which would have been mapped onto 1 by either 'two[-DIST] men' or 'two[+DIST] men' in M_4. The corresponding logical translation of this 'two' would simply be a disjunction of the logical translations of 'two[-DIST]' and 'two[+DIST]'. The denotations of [+DIST] and [-DIST] would then need to be functions which mapped the denotation of 'two' onto the denotations of 'two[+DIST]' or 'two[-DIST], or else mapped the denotation of 'two men' onto the denotation of 'two men[+DIST] or 'two men[-DIST]'.

I shall not pursue further functions which map determiners onto distributive or collective determiners. We would have problems with sentences like

(23) John and Harry ate three eggs.

(24) They ate three eggs.

which would go beyond the scope of the present study. I shall consider the possibility of DIST as an NP modifier in the next section.

(v) The Model M_5 (Distributive and Collective Readings)

We ought finally to consider whether some real synthesis of the denotations of 'two[-DIST] and 'two[+DIST]' is possible. Again we do not need to change the semantic theory substantially, but we must look carefully at the denotation of 'two'.

M_4 is a model for a language which contains the two distinct lexical items 'two[-DIST] and 'two[+DIST]'. In M_4 'two[-DIST] men' maps onto 1 any VP denotation which contains a set of two men. 'Two[+DIST] men' maps onto 1 any VP denotation which contains two singleton sets of men.

Let M_5 be a model for a language which contains the lexical item 'two', but not 'two[-DIST] and 'two[+DIST]'. The basic expressions of the language for which M_5 is a model are

VP → {ate three cakes, ate three eggs}

Det → {two}

CN → {man}

The denotations of the VPs and the CN in M_5 are as they were in M_4. In set notation, they are

ate three cakes → {{a,c}}

ate three eggs → {{a},{c}}

man → {a,c}

The denotation of 'two' in M_5 is a function which maps each of the 8 common noun denotations onto a function which maps each of the 256 VP denotations onto a truth value. This function will map onto 1 any VP denotation which contains some subset of sets of individuals, the union of which contains exactly (and only) two members from the input CN denotation. The denotation of 'two men', for example, will map onto 1 any VP denotation which contains some subset of sets of individuals, the union of which contains exactly[8] two men.

The set of sets of individuals {{a,c}}, the denotation of 'ate three cakes', will be mapped onto 1 by 'two men' in M_5, because it contains the subset of sets of individuals {{a,c}}, the union of which contains exactly two men. (Note that the subset of sets of individuals need not be a proper subset of sets of individuals.)

The set of sets of individuals {{a},{c}}, the denotation of 'ate three eggs', will also be mapped onto 1 by 'two men' in M_5, because it contains the subset of sets of individuals {{a},{c}}, the union of which contains exactly two men.

In models of the M_5 type, the denotation of 'two' is neutral with respect to collective and distributive readings. In fact it allows readings which are neither one nor the other, but is not possible to demonstrate this in so restricted a model as M_5.

Writing out a section of the function F_5 for the model M_5 would have little to offer, since it would look the same as the function suggested for M_{4b}, where 'two' would be assigned a denotation which amounted to the sum of the denotations of 'two[-DIST]' and 'two[+DIST]' in M_4. The models are too restricted for the two denotations to differ. As models of the semantics of the same simple fragment, M_{4b} and M_5 are equivalent, but they belong to separate classes of model which differ in the type of denotation admitted for 'two'.

The point will be clearer if we consider the logical translation of 'two' which corresponds to the denotation of 'two' in the class of models like M_5. This would be

(25) $\lambda Q \lambda \gamma \exists \delta (2(\cup \delta) \wedge \forall B(\delta(B) \rightarrow (\gamma(B) \wedge \forall x(B(x) \rightarrow Q(x)))))$

which would give examples like

(26) Two men ate three eggs.

$\exists \delta (2(\cup \delta) \wedge \forall B(\delta(B) \rightarrow (\text{ate three eggs}'(B) \wedge \forall x(B(x) \rightarrow \text{man}'(x)))))$

(There exists a set of sets of individuals, the union of which contains two individuals. Every set of individuals which is a member of that set of sets of individuals ate three eggs, and every member of each such set of individuals is a man.)

In this particular example, the VP has a denotation in the model M_5 {{a},{c}}, and the subject NP is distributive with respect to the VP, but we could replace the VP with 'ate three cakes', with a denotation of {{a,c}} in the model, and the subject NP would be collective with respect to the VP.

If we assume an extension of M_5 as a model for the semantics of

(15) Two men ate three cakes.

(8) One man ate three cakes.

(27) Two men ate three eggs.

(20) One man ate three eggs.

then it is possible for (15) and (27) to be true when (8) and (20) are false. The logical translations of (15) and (27) would guarantee that in each case the eating was performed by a set of individuals (subsets of the VP denotations), which does not entail that any individual performed such an action.

Although for simplicity 'slept' does not appear in the language for which M_5 is a model, there is nothing in the semantic theory assumed for M_5 which guarantees that

(28) Two men slept.

entails

(29) One man slept.

and we may not be happy about this.

It looks as though in the semantic theory assumed for M_5 we have lost all trace of distributive entailments. However, if we compare (21) and (25) (the logical translation of 'two[+DIST]' corresponding to the model-theoretic denotation of 'two[+DIST]' in M_4, and the logical translation corresponding to the model-theoretic denotation of 'two' in M_5, respectively)

(21) $\lambda Q \lambda \gamma \exists \delta (2(\cup \delta) \wedge \forall B(\delta(B) \rightarrow (1(B) \wedge \gamma(B) \wedge \forall x(B(x) \rightarrow Q(x)))))$

(25) $\lambda Q \lambda \gamma \exists \delta (2(\cup \delta) \wedge \forall B(\delta(B) \rightarrow (\gamma(B) \wedge \forall x(B(x) \rightarrow Q(x)))))$

we see that the only difference is the section '...1(B) $\wedge$...' which forces the subset of sets of individuals from the VP denotation to be a set of singleton sets of individuals.

If we introduce into the semantic framework a device which leads to '...1(B)$\wedge$...' in the logical translation in those cases where we want a distributive reading, then we can capture both collective and distributive readings.

Let us assume that the feature DIST is semantically potent on the NP rather than on the determiner 'two'. To ensure that this is so, we must include a semantic rule

Semantic Rule 3: If a member of category NP has a denotation β and the semantically potent feature [+DIST] has the denotation α, and they combine as a member of category NP[+DIST], then the denotation of that member of category NP[+DIST] is $\alpha(\beta)$.

We must also give [+DIST] a denotation in the model. ([-DIST] will be a default for any NP which does not carry the feature [+DIST], and the denotation in M_5 of, for example, the NP 'two men[-DIST]' will simply be the denotation of 'two men' already discussed.) The denotation of [+DIST] will be a function which maps NP denotations onto NP denotations. Now as we have seen, NP denotations map VP denotations onto truth values. In M_5 the denotation of 'two men' maps both {{a,b}} (the denotation of 'ate three cakes') and {{a},{b}} (the denotation of 'ate three eggs') onto 1. [+DIST] would map this NP denotation onto an NP denotation which mapped {{a,b}} onto 0 and {{a},{b}} onto 1.

All of those VP denotations mapped onto 0 by the denotation of 'two men' would also be mapped onto 0 by the denotation of 'two men[+DIST]'. Of those VP denotations mapped onto 1 by the denotation of 'two men', only those which contained two singleton sets of men would be mapped onto 1 by 'two men[+DIST]', the rest being mapped onto 0.

Let us for a moment regard the denotation of the NP 'two men' as a set of sets of sets of individuals. In the model M_5 'two men' has a denotation {{{a,b}},{{a},{b}}}, but 'two men[+DIST]' would have a denotation {{{a},{b}}}. The denotation of 'two men[+DIST]' would be a subset (of sets of sets of individuals) of the denotation of 'two men'.

The logical translation of [+DIST], corresponding to its model-theoretic denotation in M_5 will be

(30) $\lambda \Delta \lambda \gamma \exists \zeta (\forall Y(\zeta(Y) \rightarrow (1(Y) \wedge \gamma(Y))) \wedge \Delta(\zeta))$

where the upper case Greek letter Δ is a variable of the same logical type as an NP.

Let us compare the logical translations of

(31) Two men (each) ate three cakes.

associated with the models M_4 and M_5 (which now includes a denotation for [+DIST]).

Assuming the model M_4, we have seen that the logical translation of (31) would be

$$(22)\ \exists\delta(2(\cup\delta)\wedge\forall B(\delta(B)\rightarrow(1(B)\wedge \text{ate three eggs}'(B)\wedge\forall x(B(x)\rightarrow \text{man}'(x)))))$$

Assuming the model M_5, the logical translation of (31) would be

$$(32)\ \exists\zeta(\forall Y(\zeta(Y)\rightarrow(1(Y)\wedge \text{ate three eggs}'(Y)))\wedge\ \exists\delta(2(\cup\delta)\wedge\forall B(\delta(B)\rightarrow(\zeta(B)\wedge\forall x(B(x)\rightarrow \text{man}'(x))))))$$

The first line of this translation claims the existence of a set of singleton sets, each member of which ate three eggs. The second line claims the existence of a subset of this set of singleton sets, the union of which contains two individuals. The third line claims that each individual in the union of that subset of singleton sets is a man.

The set of sets of individuals claimed to exist by the first line of (32) is not a problem. If it is true that two men each ate three eggs, then there does indeed exist a set of singleton sets each member of which ate three eggs. Perhaps there are other singleton sets in this set besides the ones we are interested in, but (32) makes no claim about them. The subset of sets of individuals claimed to exist by the second line of (32) has the same properties as the set of sets of individuals claimed to exist by (22) - every set of individuals which it contains is a singular set and ate three eggs, and each singular set consists of a man, which is the distributive reading we require.

If we return now to the model M_{4b}, we can see that the logical translation which has been given to [+DIST] for models like M_5 would work also for that model. In the model M_{4b} the denotation of 'two' would be the union of the denotations of 'two[-DIST]' and 'two[+DIST]'. This would correspond to a disjunction in the logical translation of 'two men', giving a translation

$$(33)\ \lambda\delta((\exists Y(2(Y)\wedge\delta(Y)\wedge\forall x(Y(x)\rightarrow \text{man}'(x))))\vee(\exists\gamma(2(\cup\gamma)\wedge\forall B(\gamma(B)\rightarrow(1(B)\wedge\delta(B)\wedge\forall x(B(x)\rightarrow \text{man}'(x)))))))$$

Now the restriction placed on an NP denotation by [+DIST] would only be compatible with one side of this disjunction. If we apply to (33) the logical translation of [+DIST], given as (30) above, and then we apply the resulting function to 'ate three cakes', we get

$$(34)\ \exists\zeta(\forall X(\zeta(X)\rightarrow(1(X)\wedge \text{ate three cakes}'(X)\wedge((\exists Y(2(Y)\wedge\zeta(Y)\wedge\forall x(Y(x)\rightarrow \text{man}'(x))))\vee(\exists\gamma(2(\cup\gamma)\wedge\forall B(\gamma(B)\rightarrow(1(B)\wedge\zeta(B)\wedge\forall x(B(x)\rightarrow \text{man}'(x))))))))))$$

The second line is incompatible with the first, and with that line effectively disabled, the entailments of (34) match those of (32).

3. CONCLUSIONS

A number of different models and associated semantic theories have been considered for the semantics of a small fragment of English, which has been modified slightly from time to time to suit the aims of the model under consideration.

The model M_1 has VP denotations as sets, and contains the determiner 'one', but no plural determiners.

The model M_2 has VP denotations as sets. It contains the determiner 'two'. It can capture distributive readings, but not collective readings.

The model M_3 has VP denotations as sets of sets. It contains the determiner 'two'. It can capture collective readings but not distributive readings.

The model M_4 has VP denotations as sets of sets. It has the determiners 'two[+DIST]' and 'two[-DIST]', and is able to capture distributive and collective readings.

The model M_{4b} is like the model M_4, except that it has the single determiner 'two' which has as a denotation the union of the denotations of 'two[+DIST]' and 'two[-DIST]'.

The model M_5 has VP denotations as sets of sets. It has the determiner 'two', and is able to capture distributive and collective readings.

In models M_4 and M_5 the distributive and collective readings would have distinct model-theoretic denotations (and logical representations) which would guarantee the sorts of entailments we first considered with sentences (1), (2) and (3). The different readings would also be syntactically distinct, marked by different values of the feature DIST.

There has been a progression in the models presented here, with each able to handle more data than the models which preceded it, at least up to model M_4. Model M_5 has a more general denotation for numbers than models M_4 and M_{4b} (it will capture readings which are neither entirely distributive nor entirely collective), but it is not obvious that this would more appropriate for a larger grammar of English.

There are a number of outstanding issues which are not discussed here (further discussion may be found in Knowles 1994). We may wish for example to consider distributivity with respect to groups rather than individuals. Landman identifies a reading for

(35) The boys and the girls meet (but not in the same room). (Landman 1989 page 591) where the subject NP 'the boys and the girls' is distributive with respect to the two groups (each group meets separately), but collective with respect to each group (each group meets together as a group). A more complex model than any I have discussed would be required to capture such readings.

The models presented here have NPs as functors on VPs, and this is by no means a universally accepted way of modelling natural language. A number of reasons are given in the literature (e.g. Dowty and Brodie 1984 page 76, Gazdar et al. 1985 pages 189-192) for preferring a VP-as-functor analysis.

We may wish to consider the need for a separate denotation for [-DIST] to guarantee collective entailments. I am personally doubtful about the need for such a denotation (but see Landman 1989 page 597).

We certainly would need to consider the role of the feature DIST in the syntax. How could we guarantee for example that a VP such as 'slept' always has a subject NP which carries the feature [+DIST]? This may be straightforward for a VP such as 'slept', but how could we guarantee it for a VP such as 'won a 100 metre dash' (see Roberts 1987 page 6)?

The models proposed here represent only a very tiny corner of the semantics of a natural language. Their worth must be judged in relation to models of greater complexity than I have been able to consider here.

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1 In what follows, what I refer to as a set may sometimes be shown as the characteristic function of that set, so that

$$\begin{bmatrix} a \rightarrow 1 \\ b \rightarrow 0 \\ c \rightarrow 1 \end{bmatrix}$$

and {a,c} are taken to be equivalent. Dowty, Wall and Peters (1981 page 26) discuss the isomorphism between the two sorts of objects.

2 Bennett uses present simple tense but I shall use past simple tense, to avoid the question of verb agreement.

3 Neither PTQ nor Bennett's first fragment has a syntactic category of determiner. Determiners are individually introduced in separate syntactic and semantic rules.

4 I have chosen to include 'one' rather than 'a(n)' simply because the only determiners I shall consider will be numerals, which provide useful insights into distributive

entailments. I have no reason to suppose that the comments which I shall make about the semantics of numerals cannot, where relevant, apply equally to other determiners.

5 (11) and (12) together are no simpler than (9) which they replace, but it is convenient to have a notation which uses the relevant numeral in the logical translation of the number word.

6 It is not a problem that they should share the same denotation in these very restricted models. In the real world there are many properties which we might associate with the property of being a man, but admittedly the property of having eaten three cakes is not one which immediately springs to mind. In these models, every individual who is a man happens to have eaten three cakes, and every individual who ate three cakes happens to be a man.

7 I have deliberately avoided the complication of the 'at least' and 'exactly' readings, but here on an 'exactly one' reading of (20), (19) should not entail (20). The model could be adapted to include this distinction, but as discussed above it is not my main concern, and I shall assume that it is accounted for by Horn's 'rules of conversation' (Horn 1976 pages 31-32).

8 The denotation of 'two' in M_5 is in fact the denotation of 'at least two'. Provided a VP denotation has a subset of sets of individuals, the union of which contains exactly two men, then that VP denotation will be mapped onto 1 by the function which is the denotation of '(at least) two men', regardless of whether or not the VP denotation contains any other sets of individuals which contain men or other individuals. The inclusion of 'exactly' in the paraphrase for 'at least two' ensures for example that 'Two men carried a piano' does not come out true in circumstances in which five men carried a piano together. The denotation of 'exactly two men' in M_5, if we felt the need for such a denotation, would map onto 1 only those VP denotations which had a subset of sets of individuals, the union of which contained exactly two men, but which contained no other men in the union of the VP denotation.